X-rays
and
Computed Tomography (CT)

Outline

X-rays

- Basic principle
- Generation of x-rays
- Interaction with body
- Applications and specialized techniques

Computed Tomography (CT)

- Basic principle
- Projection, sinogram, reconstruction
- Generations of CT scanners
- Applications

X-rays

The beginnings of radiology

- X-rays discovered in 1895 by **Wilhelm Röntgen**
  - While experimenting with *cathode tubes* in a dark room, he forgot a *metal plate* on the bench
  - He noticed a faint *fluorescent glow* emanating from it, so he went to remove the plate
  - He saw the image of *his hand's bones* cast onto the plate!

The beginnings of radiology

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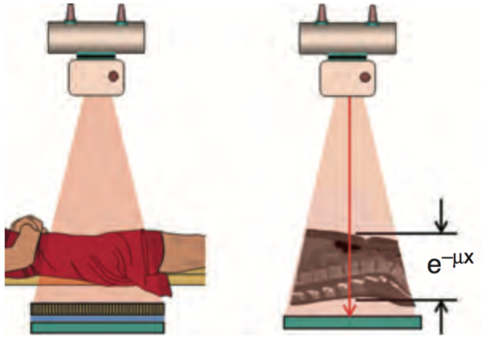
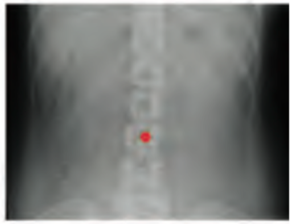
- First medical x-ray image
  - "*Hand mit Ringen*" (Hand with Rings)

- First **Nobel Prize in Physics** in 1901
  - Röntgen <u>refused to patent</u> his discovery: he wanted all mankind to benefit from practical applications of x-rays
- Father of radiology

Basic principle

- An x-ray beam passes through the patient
  1. Generated with the **x-ray tube**
  2. X-rays **interact** with body
  3. A **film/detector** placed just below the patient collects residual x-rays





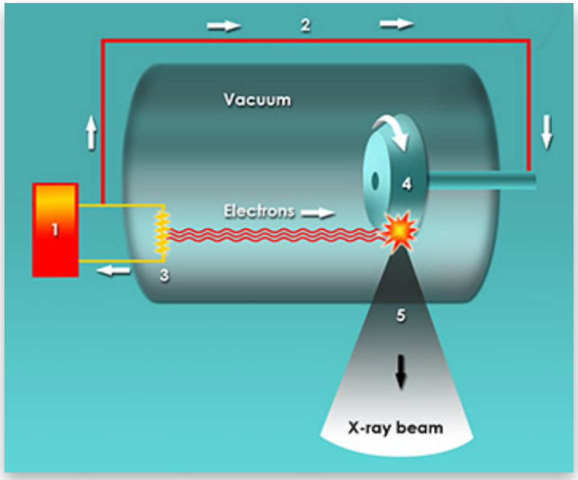
- X-rays interact differently with different tissues
  - *Bones* absorb more than soft tissues
  - Air almost does not interact (e.g. *lungs*)

- Notes:
  - Very *high contrast*
  - Resulting image is *planar projection* (2D)
  - *Ionizing radiation* (dangerous!)

X-ray generation process (1/2)

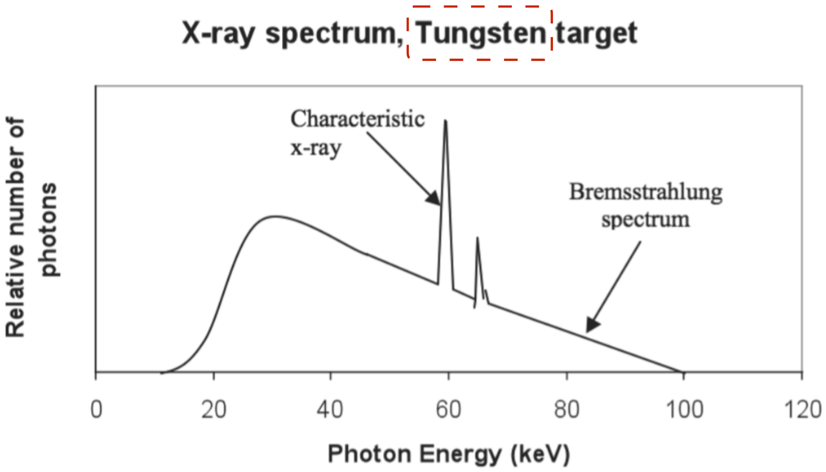
The x-ray tube

- Potential difference between *cathode* (1) and *anode* (4)
- Electrons are *accelerated* and gain energy
- When they hit the metal surface in the anode, part of their *kinetic energy* is converted to *x-ray*
- Interaction of high-energy electrons with metal nuclei
- Notes
  - vacuum required to avoid unwanted interaction (lose energy)
  - converted energy: 1% x-rays, 99% heat
  - *anode rotates* to dissipate the heat



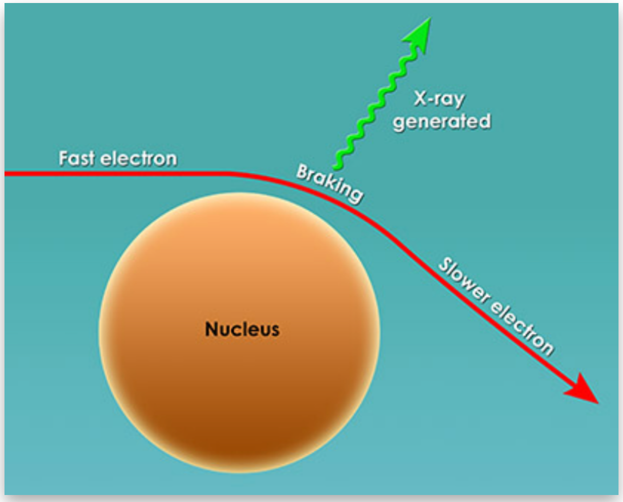
Energy spectrum

- X-ray beam contains photons with broad range of energy
- Each metal has characteristic spectrum



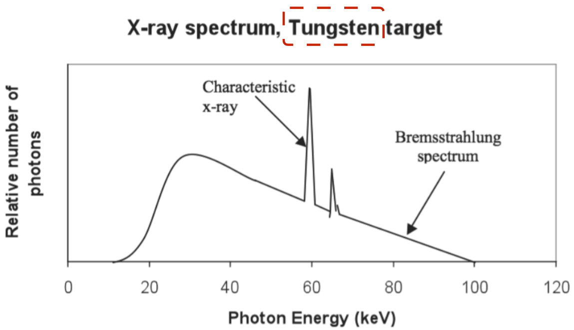
Breaking radiation

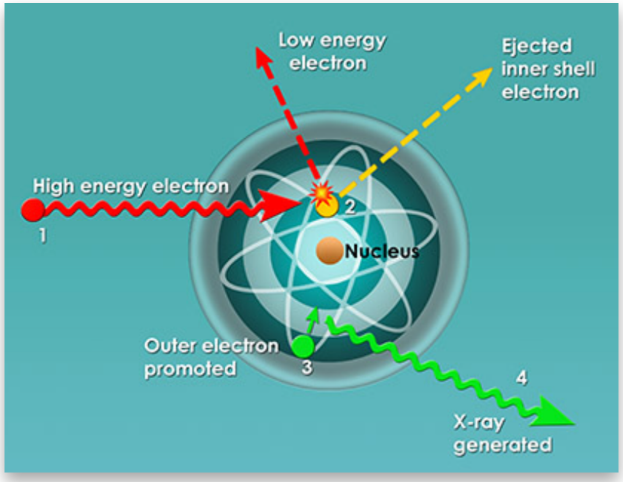
- High-energy electrons are deflected by nuclei (positively-charged)
- Kinetic energy is lost and converted to x-rays



Characteristic radiation

- **Collisions** between high-energy electrons and electron in the metal nuclei
- Lower probability, lower number of photons
- *Energy peaks* are characteristic of metal used

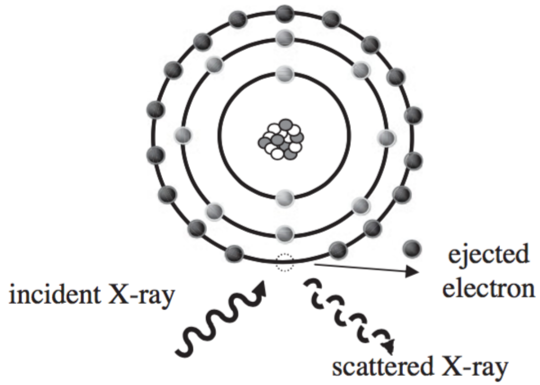


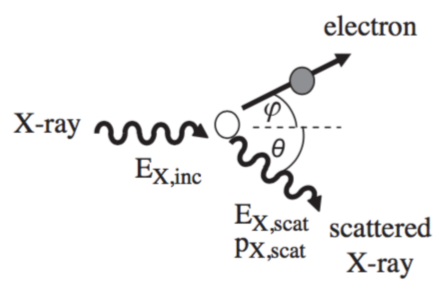


Interaction of x-rays with body (1/2)

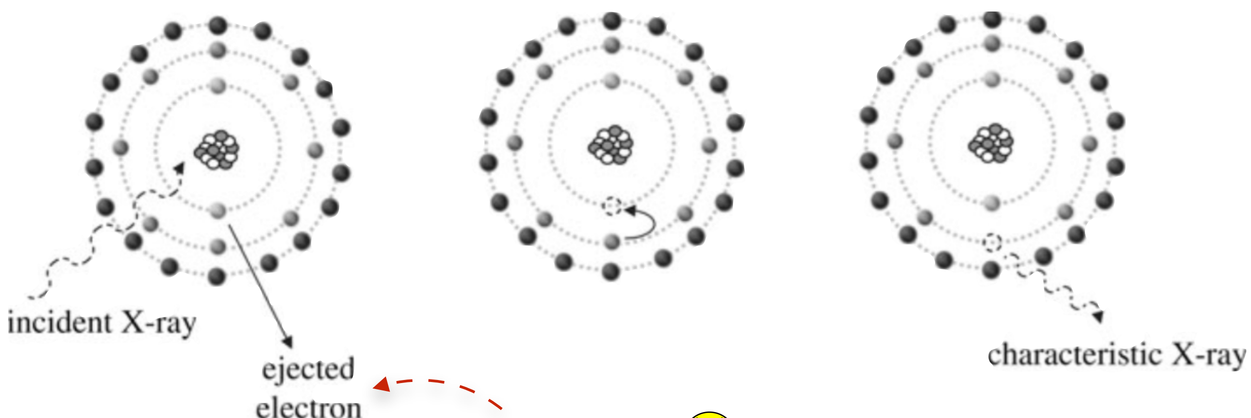
Similar principle of x-ray generation

- Compton scattering





- Photoelectric attenuation

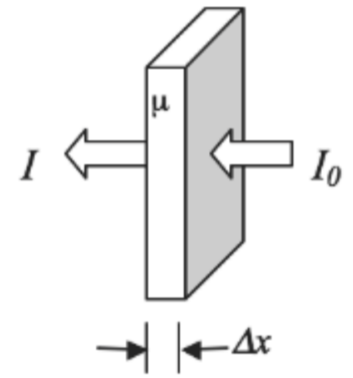


ionizing radiation

Interaction of x-rays with body (2/2)

Simple case: single material

- Beam intensity (I_0) is *attenuated* by absorption/deflection
- Every material has specific *attenuation coefficient* (μ)
- Attenuation directly proportional to *material thickness* (Δx)
- Final intensity (I) is given by *Lambert Beer law*:



$$I = I_0 e^{-\mu \delta x}$$

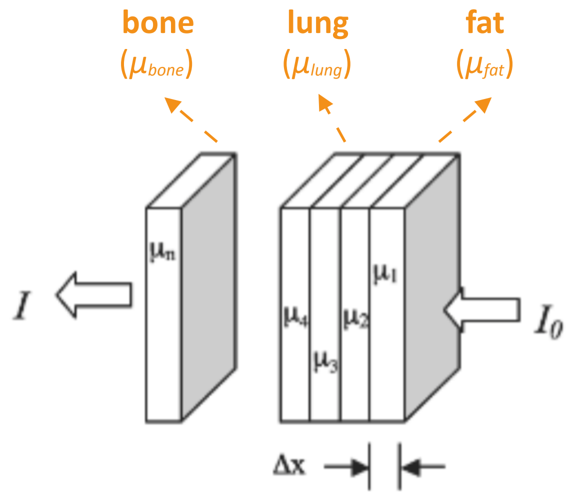
Real case: multiple materials

- Lambert-Beer law applied in *cascade*:

$$I = I_0 e^{-\mu_1 \Delta x} e^{-\mu_2 \delta x} \ldots e^{-\mu_n \delta x} = I_0 e^{-\sum_{i=1}^{n} \mu_i \Delta x}$$

- Usually, *attenuation ratio* (p) is reported:

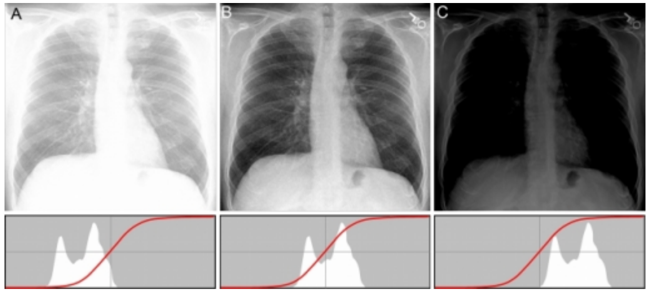
$$p = -\ln\left(\frac{I}{I_0}\right) = \sum_{i=1}^{n} \mu_i \Delta x = \int \mu(x) dx$$



Equipment

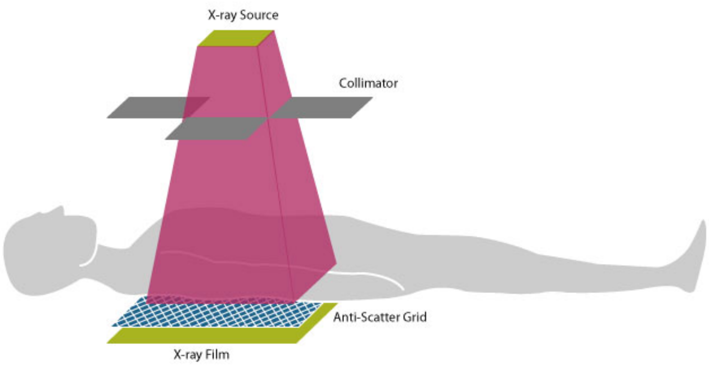
Detectors

- *Traditional film* vs *digital detectors*
- Possibility to adjust contrast



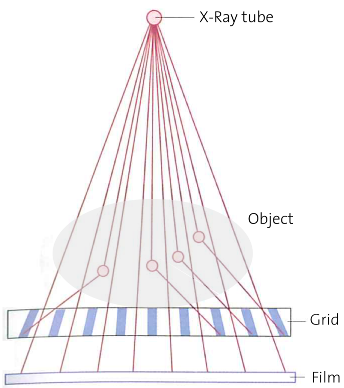
Collimators

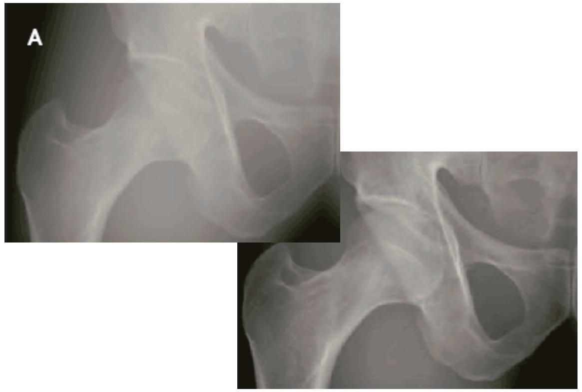
- Want to image a specific FOV
- Reduce *radiation dose* (unnecessary)
- Limit *scattered x-rays* from outside FOV



Anti-scatter grids

- Scattered x-rays reduce CNR
- Limit this *secondary radiation*
- NB: reduce also image intensity

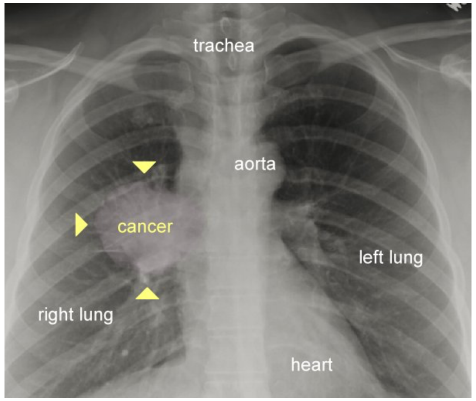




Applications and specialized techniques (1/2)

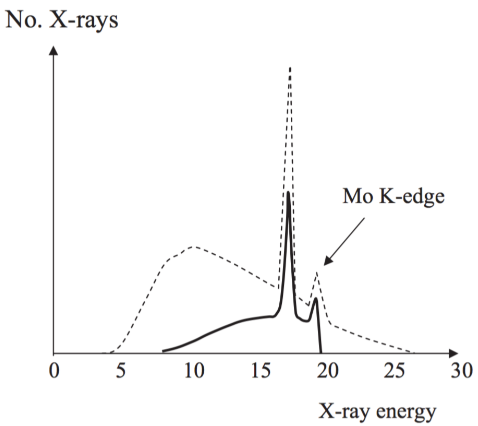
Main applications

- Fractures/cracks in bones
- Lung cancer



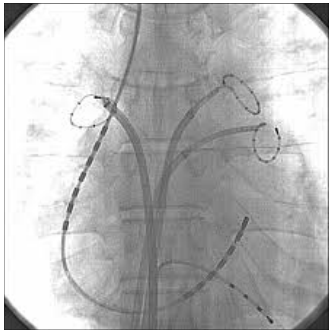
Mammography

- Detect *small tumors* or *calcifications* in the breast
  - less than 1 mm
- NB: **low radiation dose** to avoid tissue damage
- Specialized anode made of *molybdenum*



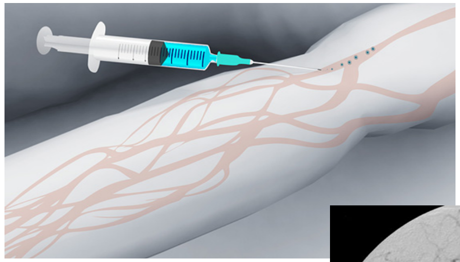
Applications and specialized techniques (2/2)

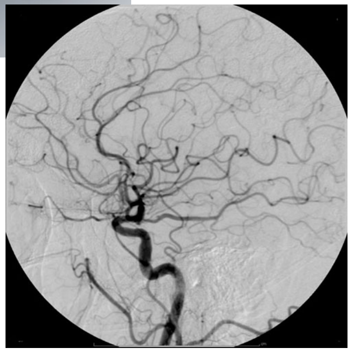
Fluoroscopy



- Placement of catheters, guide-wires etc
- *Continuous x-ray imaging*
  - very short x-ray pulses (5-20 ms)
  - 30 frames/s
- Special equipment to reduce x-ray dose

Angiography



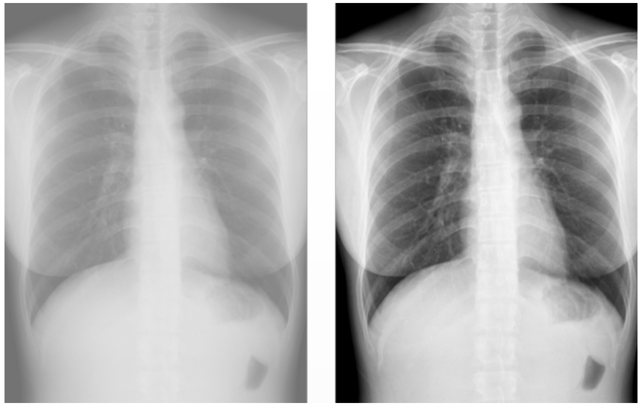


- High-resolution images of *vasculature*
- Procedure:
  - acquire a regular image
  - another after injecting a *contrast agent* into the bloodstream
  - subtract the two images
- Contrast agent
  - chemical substance that *accumulates* in specific organs/tissues
  - enhances the contrast of that structure (w.r.t. other tissues)

Image quality

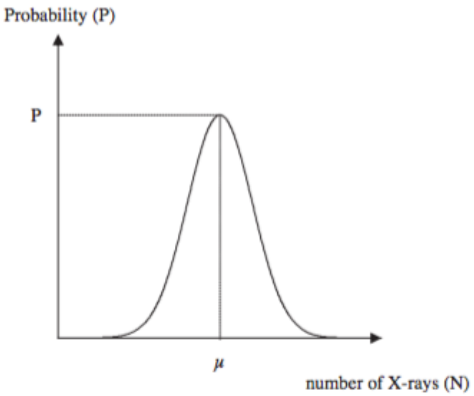
Artifacts and post-processing

- There's no actual post-processing here
- Most artifacts due to:
  - calibration/exposure
    (e.g. over- and under-exposure)
  - hardware failure
    (e.g. detectors)
  - operator/patient mistakes
    (e.g. movement, necklaces, finger marks etc)



Note

- Noise in Poisson distributed
  - mean = μ
  - *std* = $\sqrt{\mu}$
- $\text{SNR} \propto \sqrt{\text{N}}$
- SNR 2x $\longrightarrow$ radiation dose 4x



Summary

Pros

- Quick (< 2-3 *s*) and accurate (0.1 *mm*)
- Cheap

Cons

- Ionizing radiation ☢
- 2D projections

Applications

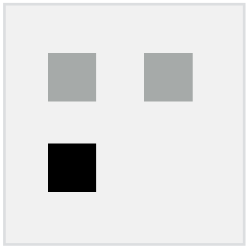
- Bones fractures
- Calcifications
- Lung cancer

Computed Tomography

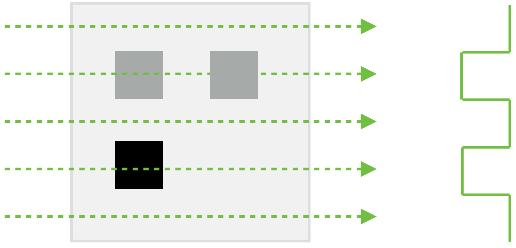
Basic principle

Problem: X-ray imaging is planar

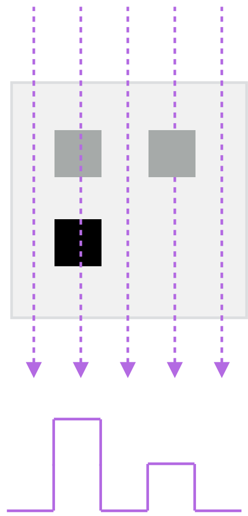
- 2D images from 3D object
- Structures can overlap and be hidden



object to be studied



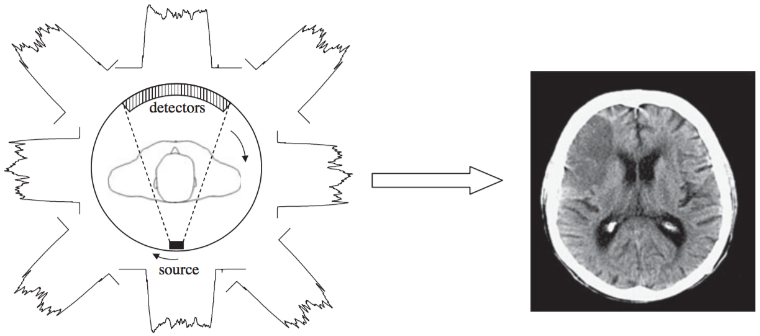
single x-ray acquisition

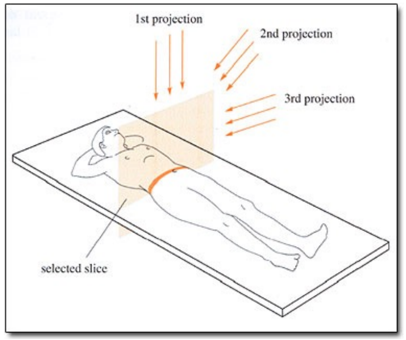


single x-ray acquisition

Solution: Computed Tomography (CT)

- Idea is to acquire many **projections**
  - *Tomography* = "picture of a plane”
  - *Computed* = “need reconstruction”

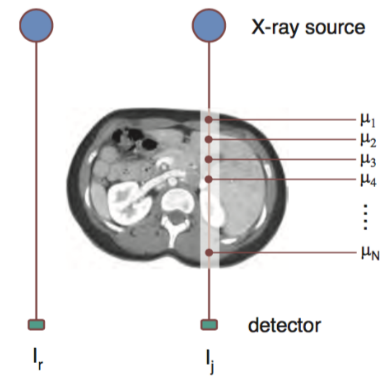




Projection coordinate system

Assumption: X-rays are parallel

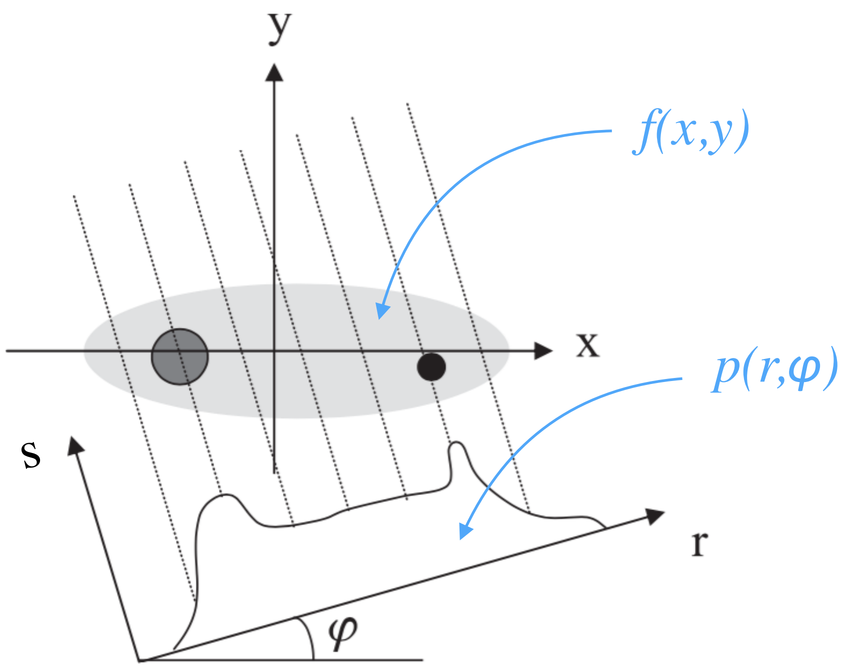
- Corresponds to *1st generation scanners* (see later)
- Simplifies math, to get the idea



Projection: new coordinate system

$$\begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- $f(x,y)$ is the *object* to study
- $p(r,\varphi)$ is the acquired projection
- φ is the *angle* at which the projection was acquired
- s represents the coordinate along the X-ray

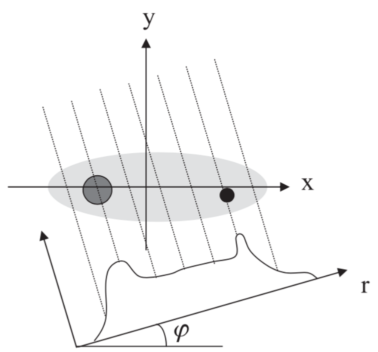


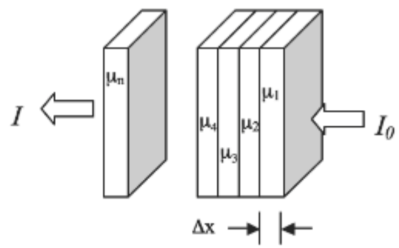
Hounsfield units (HU)

Recall:

- Intensity measured at detector *r*:

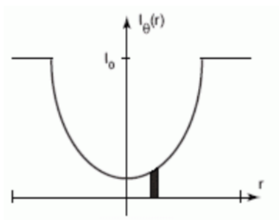
$$I_\varphi(r) = I_0 e^{-\int_{\text{line}} \mu(x,y)ds}$$
$$= I_0 e^{-\int_0^\infty \mu(r\cos\varphi - s\sin\varphi, r\sin\varphi + s\cos\varphi)ds}$$

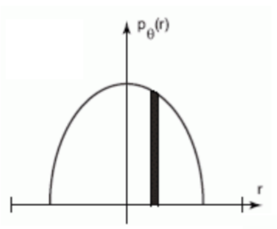




- Attenuations are always reported:

$$p = -\ln\left(\frac{I}{I_0}\right)$$





Moreover, in CT, pixel values are normalized w.r.t. water

$$\text{CT}(x,y) = 1000\,\frac{\mu(x,y) - \mu_{\text{water}}}{\mu_{\text{water}}}$$

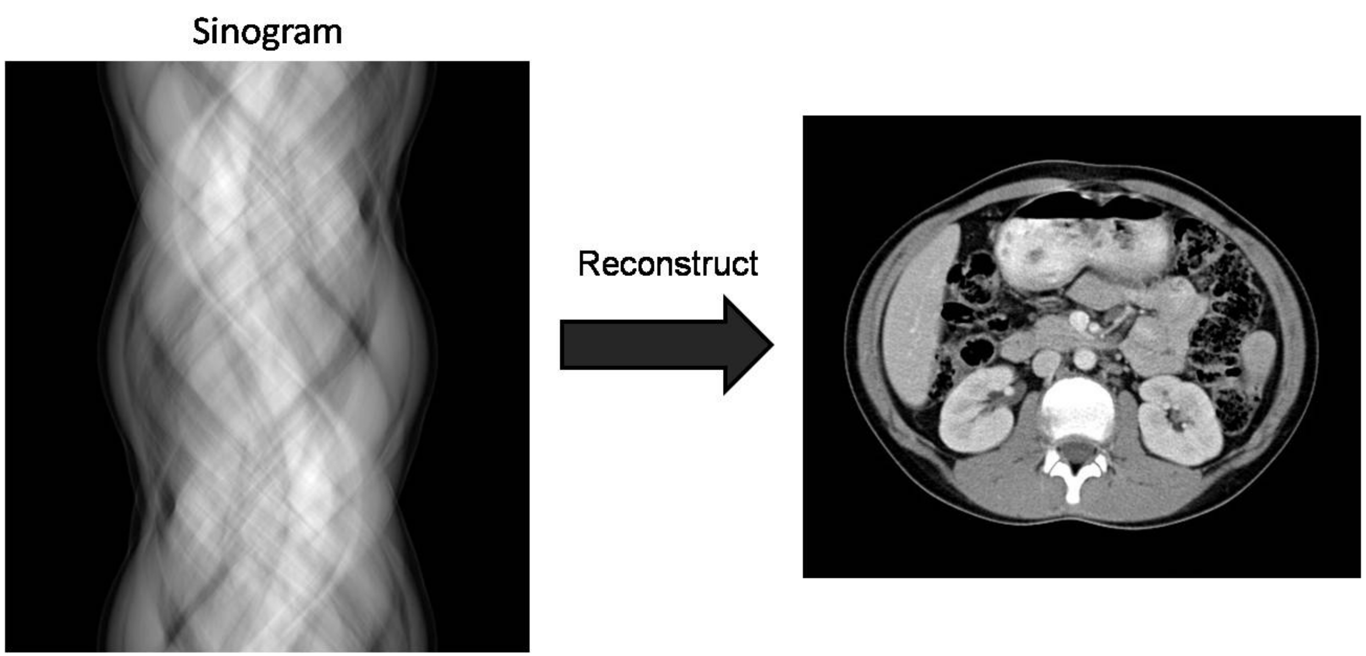
- Hounsfield units (HU)
- Values comparable between scanners

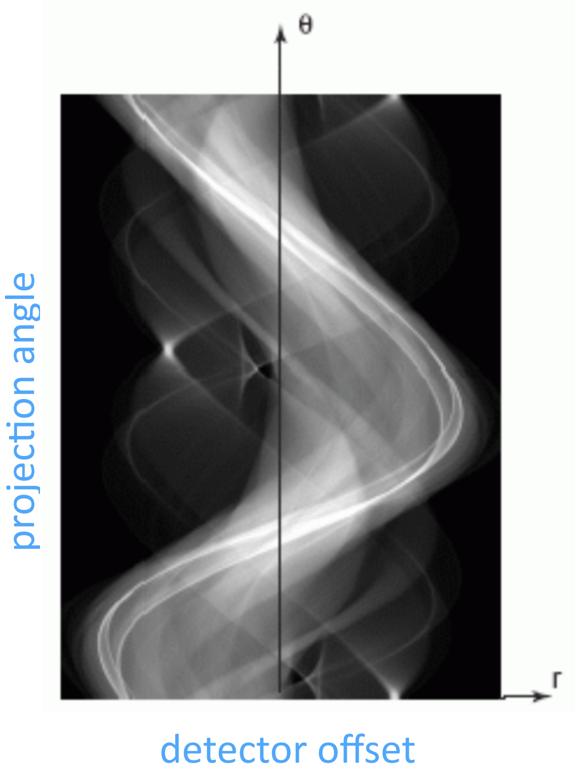
Tissue	CT number (Hounsfield units)
Bone	1000–3000
Muscle	10–40
Water	0
Lipid	−50 to −100
Air	−1000
Brain (white matter)	20 to 30
Brain (grey matter)	35 to 45
Blood	40

Sinogram

- **Collection of projections** at several angles
  - i.e. $p(r,\varphi)$, it's a 2D image

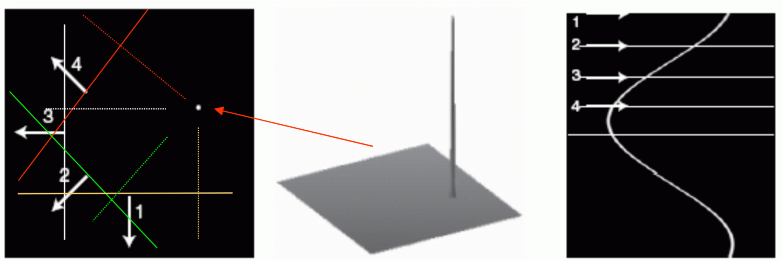
- NB: this is the data that is **actually acquired** by the CT scanner!!!



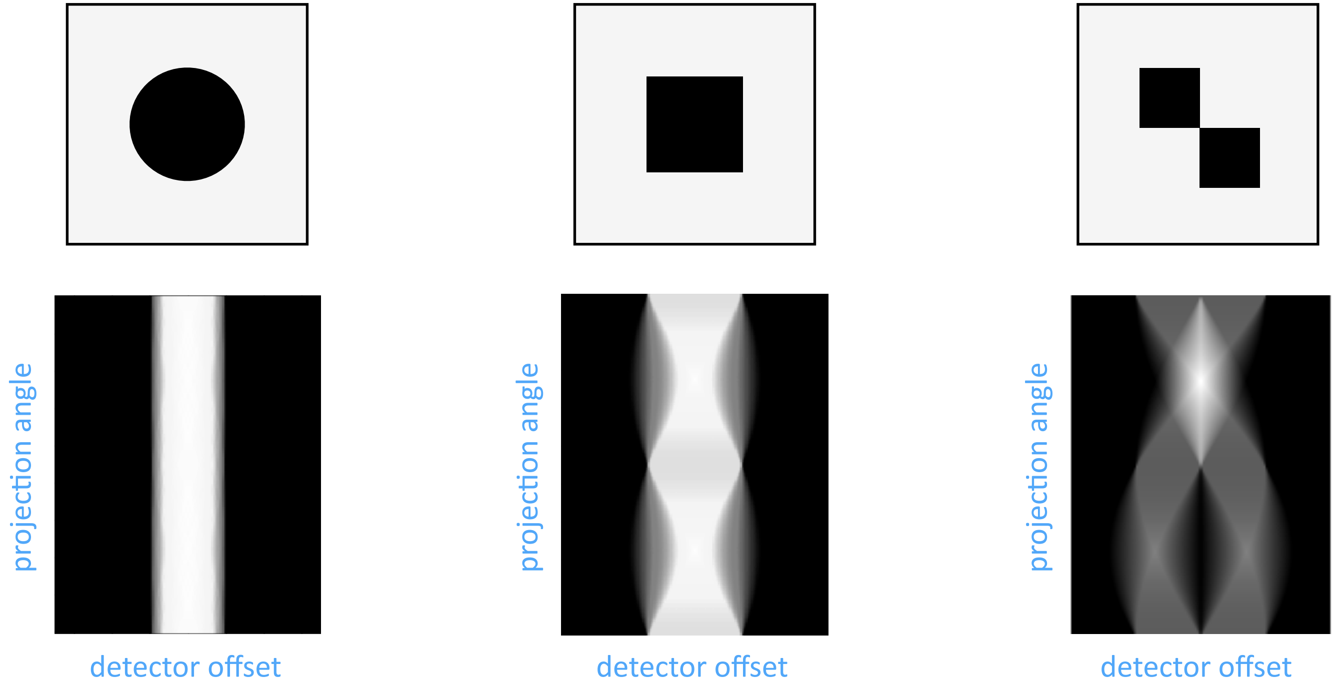


Basic example

- Imagine a simple object, i.e. *single point*
- As *system rotates* and projections are acquired...
- ...and its signal describes a *sinusoid*!

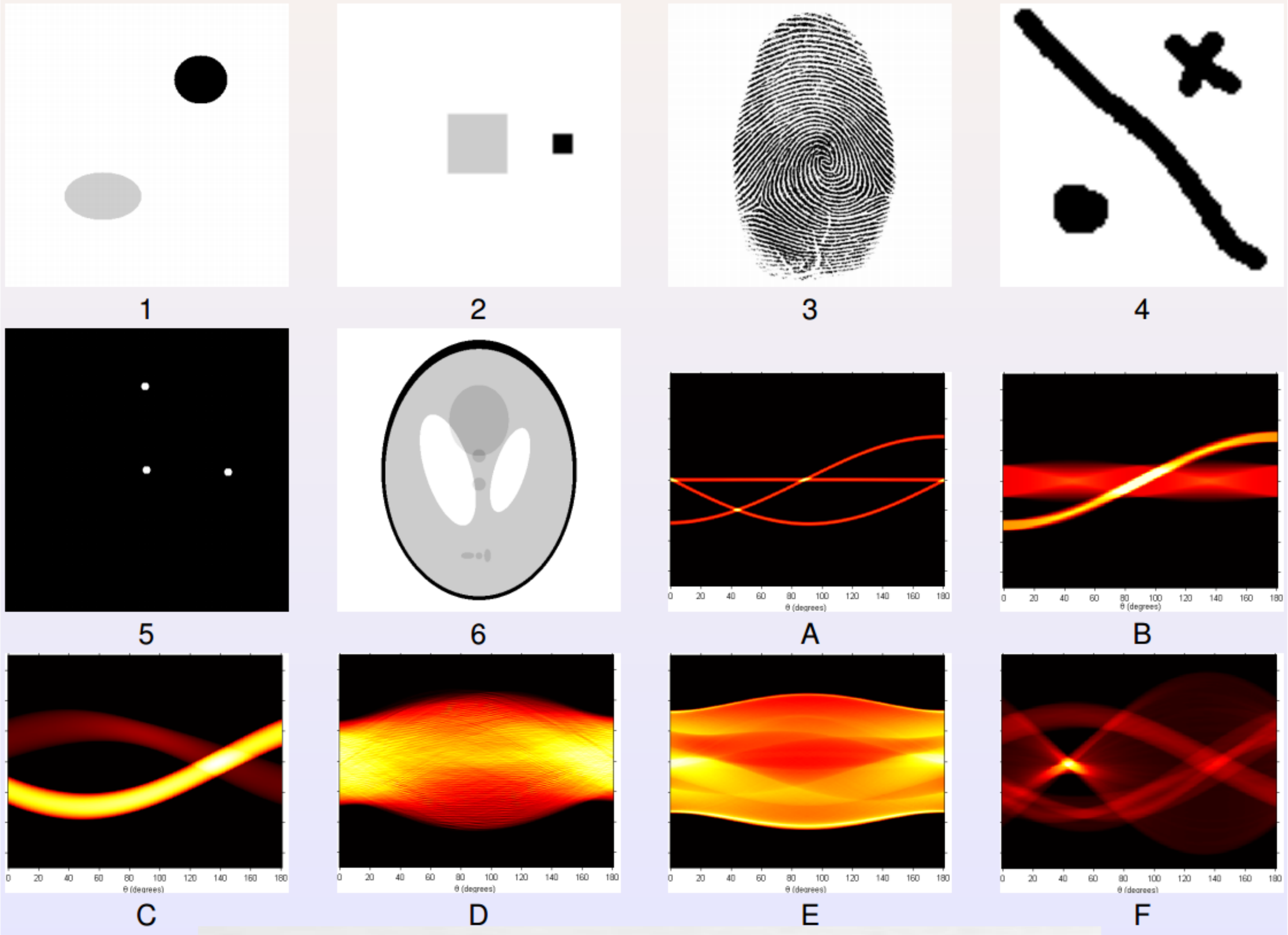

projections point object sinogram

Other examples



Sinogram

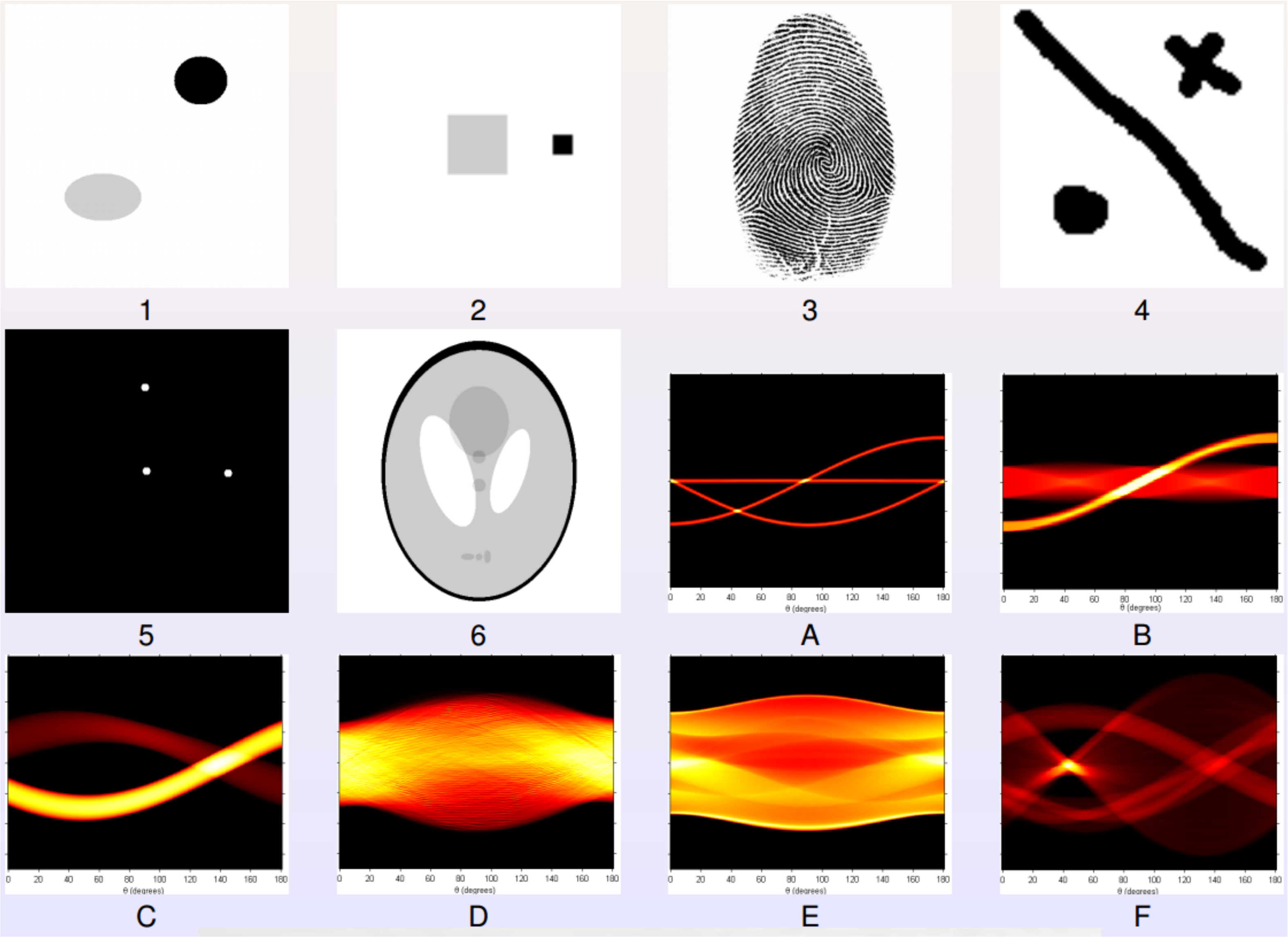
- Can you match *objects* with *sinograms*?



Sinogram (3/3)

- Can you match *objects* with *sinograms*?

1→C, 2→B, 3→D, 4→F, 5→A, 6→E



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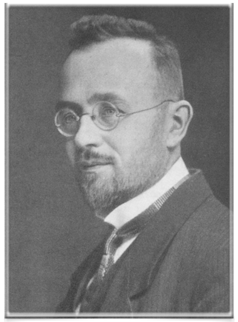
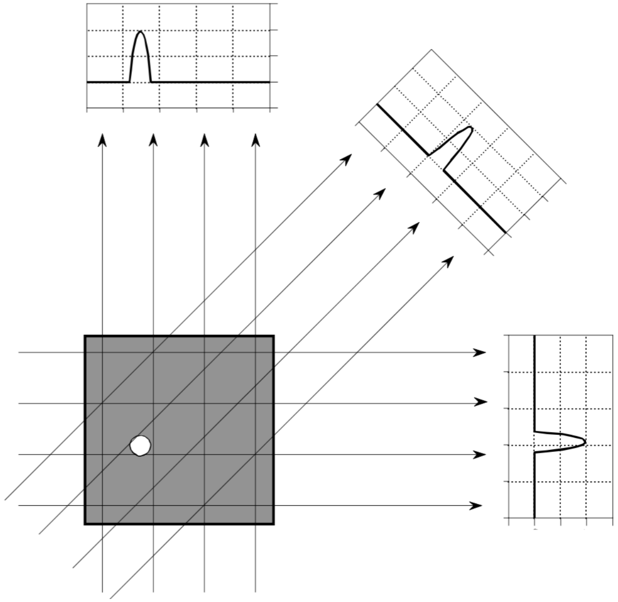


- Introduced in 1917 by **Johann Radon**
  - Transformation from **image** $f(x,y)$ to its **sinogram** $p(r,\varphi)$:

$$p(r,\varphi) = R\{f(x,y)\}$$
$$= -\int_{-\infty}^{\infty} f(r\cos\varphi - s\sin\varphi, r\sin\varphi + s\cos\varphi)ds$$

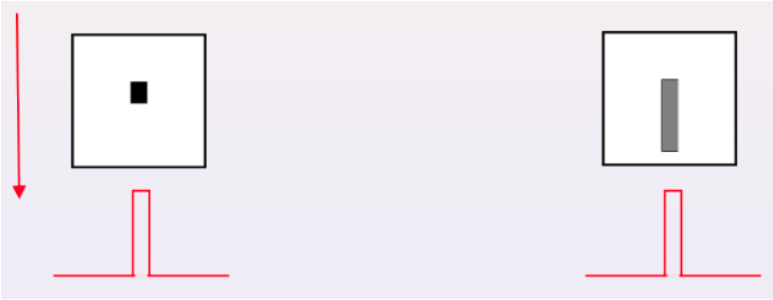


- He also provided a formula for the **inverse transform**
  - We will see some algorithms in a second
  - We will implement it in one lab

- Used by **G. Hounsfield** and **A. Cormack** as the base for CT reconstruction in 1972
  - Jointly awarded for **Nobel Prize in Medicine** in 1979

The Radon transform (2/2)

- Radon showed that it is possible to reconstruct an object for which **all its projections are known**
  - In practice, only a finite number are available (i.e. trade-off with patient comfort)
  - Multiple solutions are always possible/compatible!



1 projection

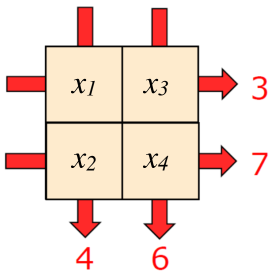
2 projections

- Always a **bias** in the reconstructions
  - Advanced reconstruction methods, e.g. iterative techniques (ART)
  - Introduce prior knowledge, e.g. spatial regularization

Reconstruction: system of linear equations

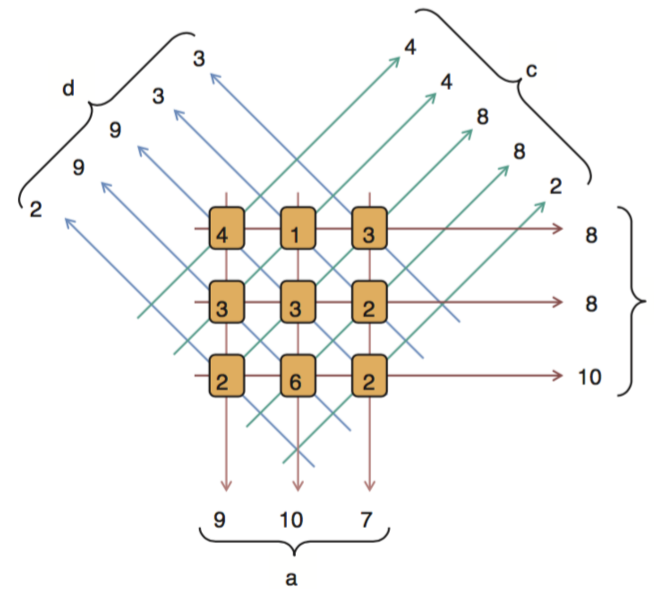
Totally impractical, but gives an idea of the problem

- One equation can be written for each measurement
  - a particular sample in a particular profile is the *sum of a particular group of pixels* in the image



$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 4 \\ 6 \end{pmatrix}$$

- To calculate N^2 unknowns (all pixels in the image) there must be N^2 independent equations
  (i.e. N^2 measurements are required)

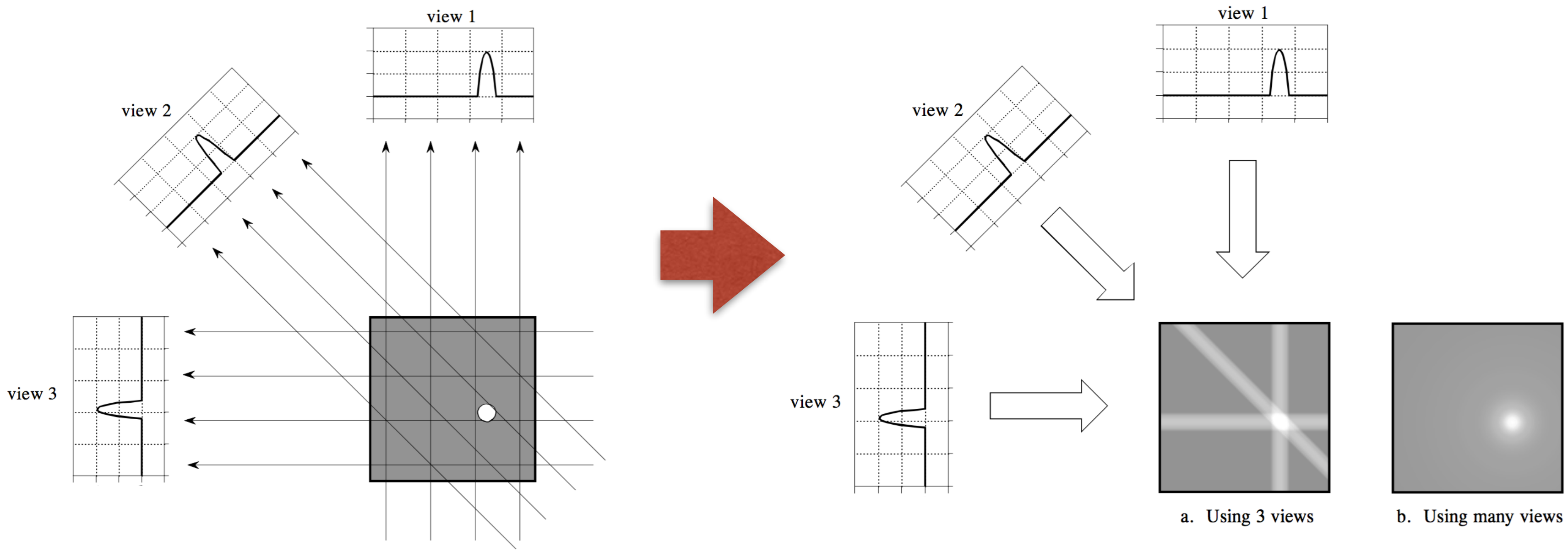


Process is similar to “Sudoku”

Drawbacks

- **Computation time**: several hundred thousand simultaneous equations
- **Stability** (for the same reason)

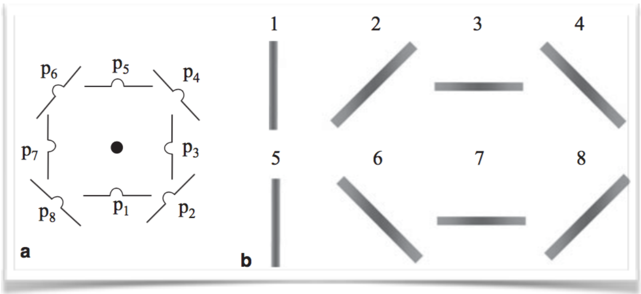
- It's the most **intuitive approach**
  - Take each view and **smear it back** along the path it was originally acquired



- The **reconstructed image** is given by:

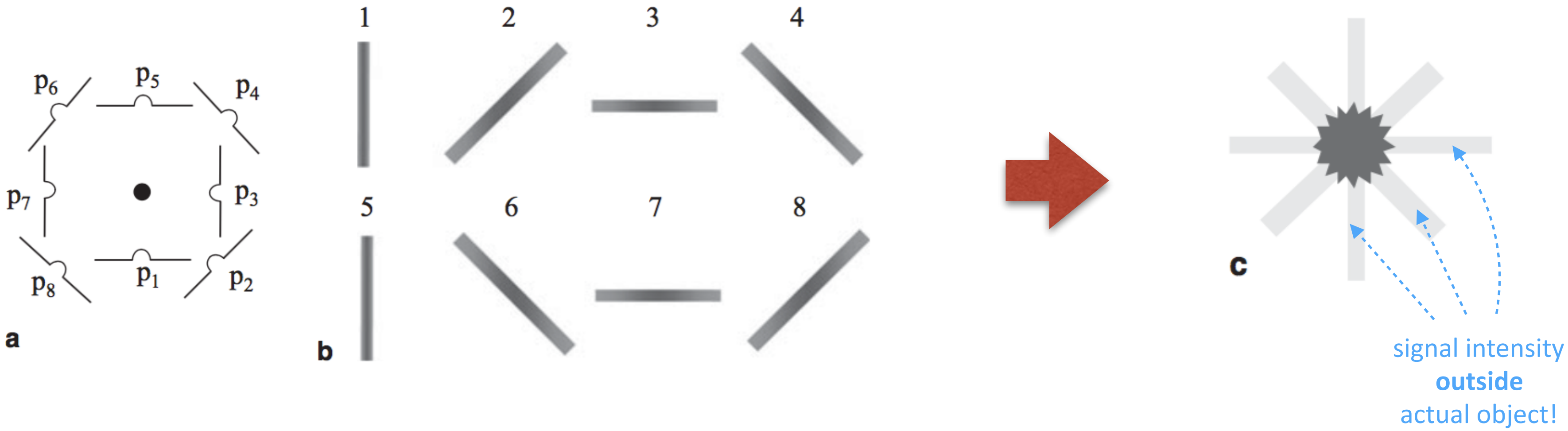
$$\hat{f}(x,y) = \sum_{i=1}^{n} \tilde{p}(r,\varphi_i)\Delta\varphi$$

estimated image

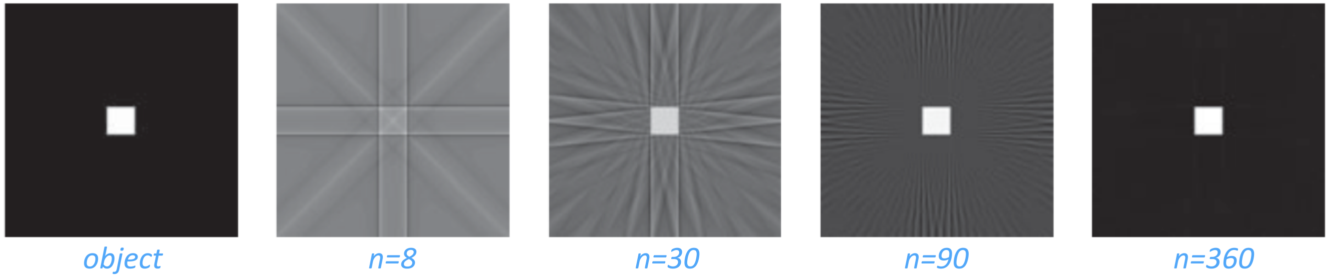


smearing process

- **"Star effect"**: resulting image is **blurred**



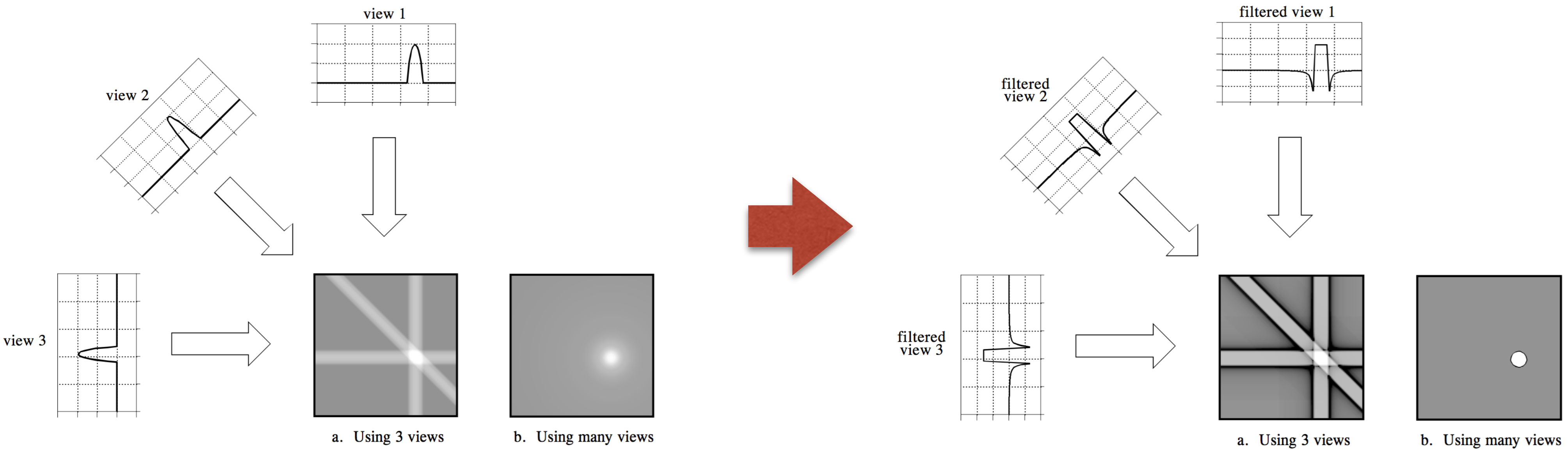
- Artifact **can be reduced** by acquiring more projections



  - Even a large number of projections leads to blurring
  - More projections ➜ longer scan time, i.e. patient comfort
  - More projections ➜ more radiations ☢

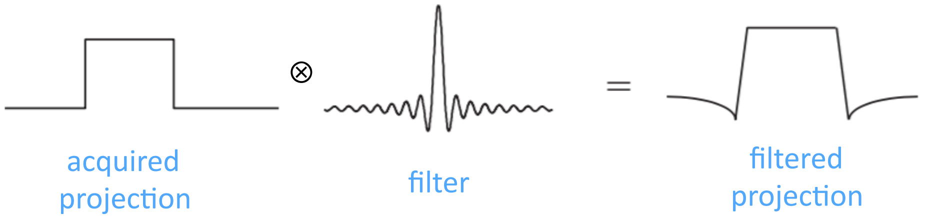
It is a **well-understood** artifact

- <u>Fourier theory</u>* shows that this blurring can be reduced by applying an **appropriate filter function** to each projection before reconstruction



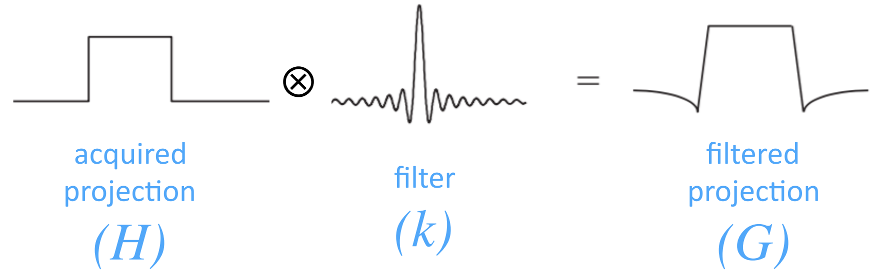
Intuition

- "**Negative lobes**" introduced around the edges **cancel out** the unwanted "**positive blurring signal**" around the original edges of the object



Reconstruction: filtered back-projection (2/3)

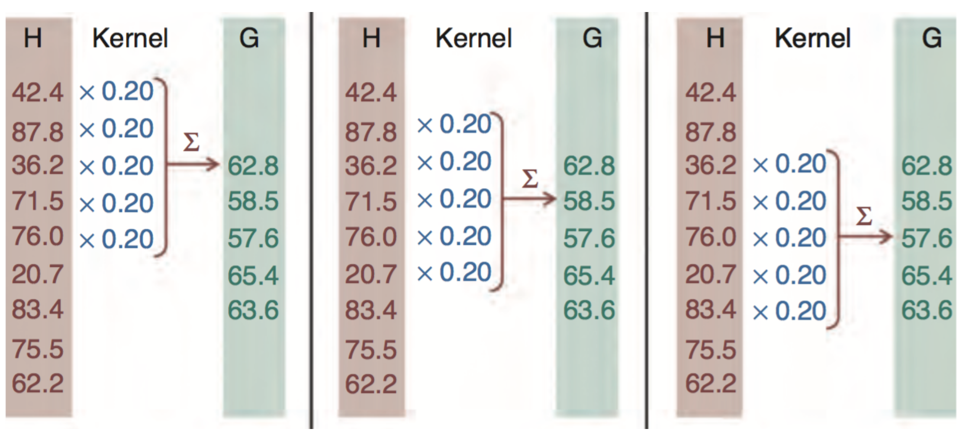
- The filter is applied via **convolution**



defined as

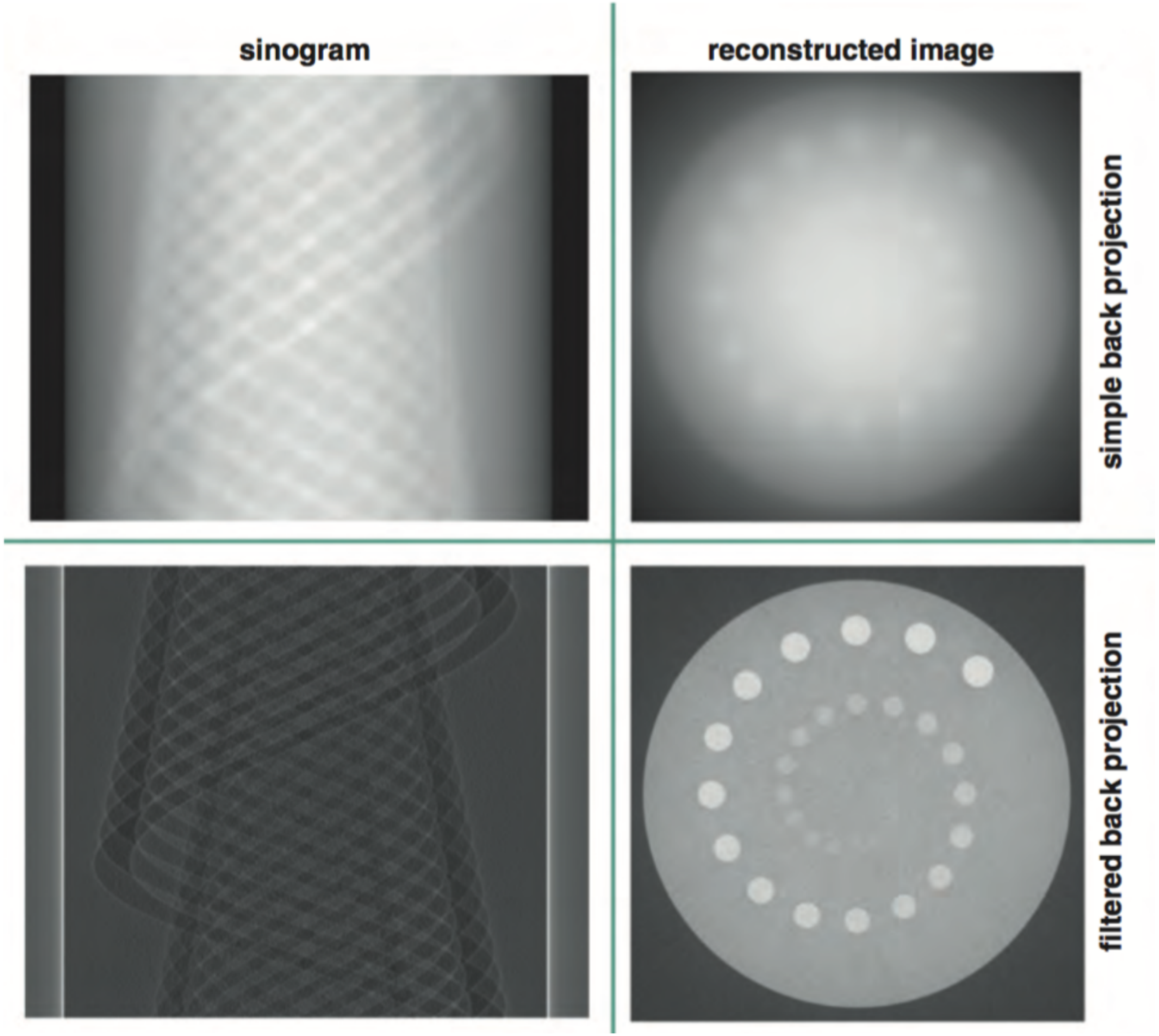
$$G(x) = \int_{-\infty}^{\infty} H(x')\, k(x - x')\, dx' = H(x) \otimes k(x)$$

- In practice…



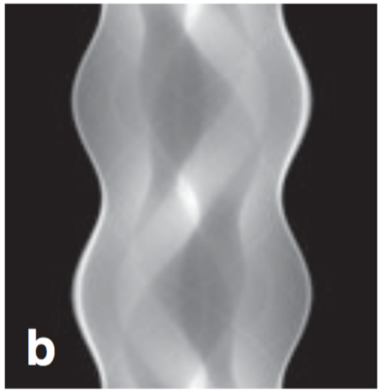
- Note
  - This operation is really time consuming (**very slow**!)
  - Much more efficient if performed in Fourier space*

- *Simple* vs *filtered* back-projection



- *Simple* vs *filtered* back-projection

object

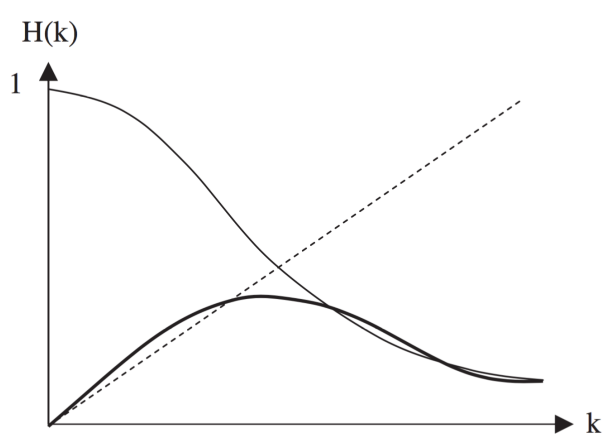


sinogram

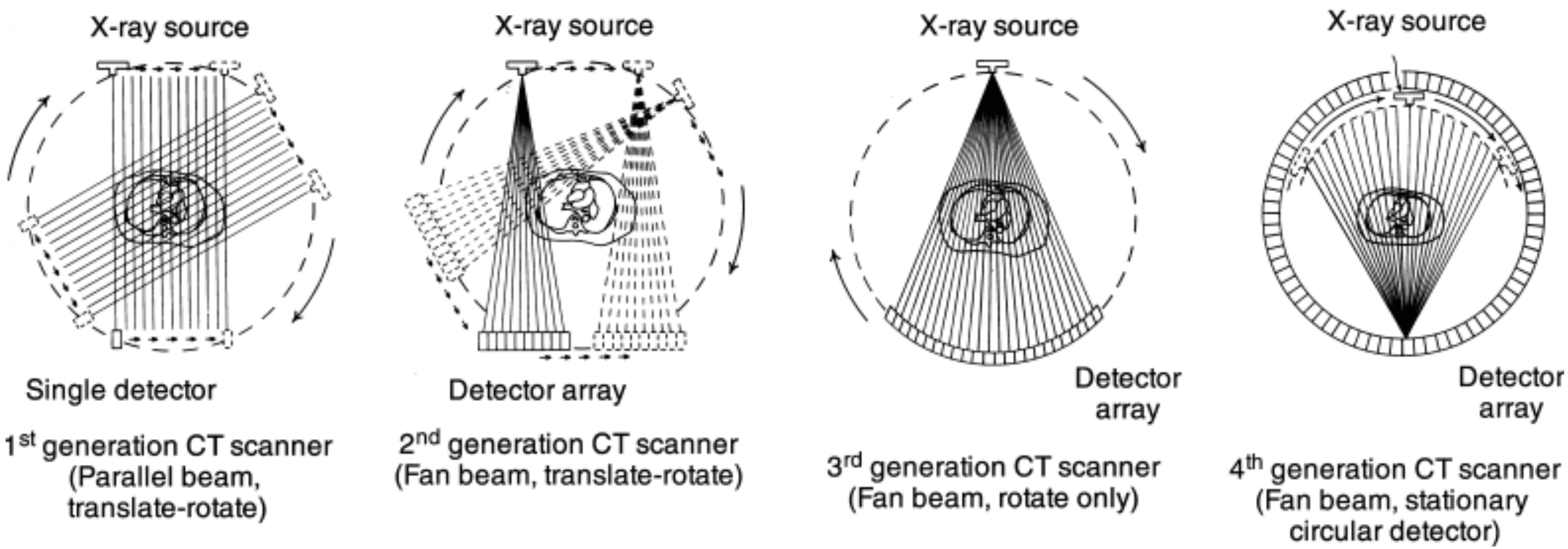
simple back-projection

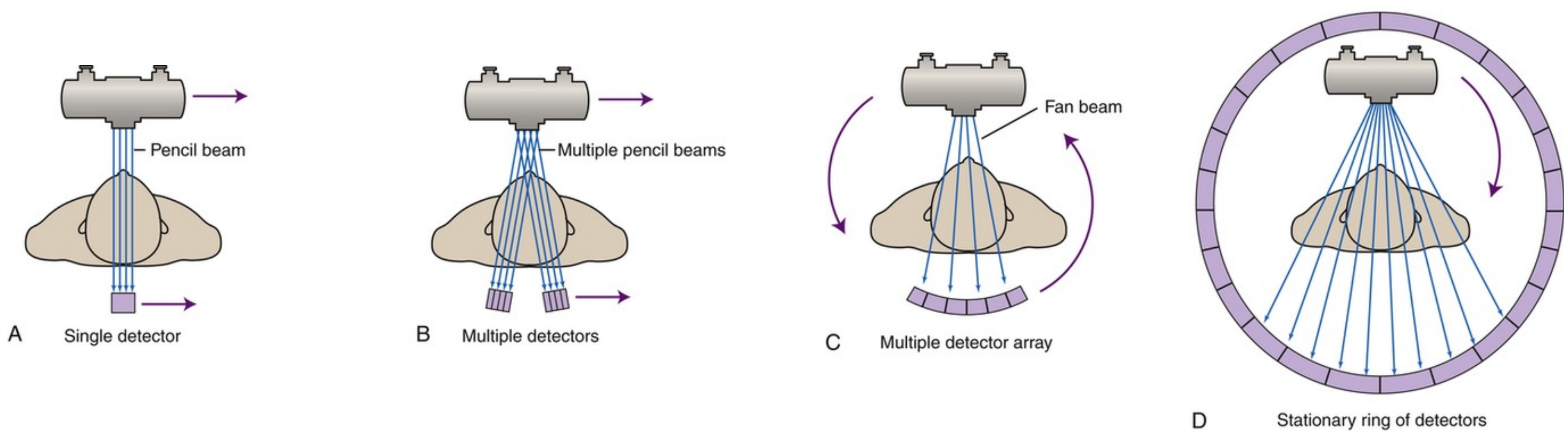
filtered back-projection **(filter 1)**

filtered back-projection **(filter 2)**



Scanner generations

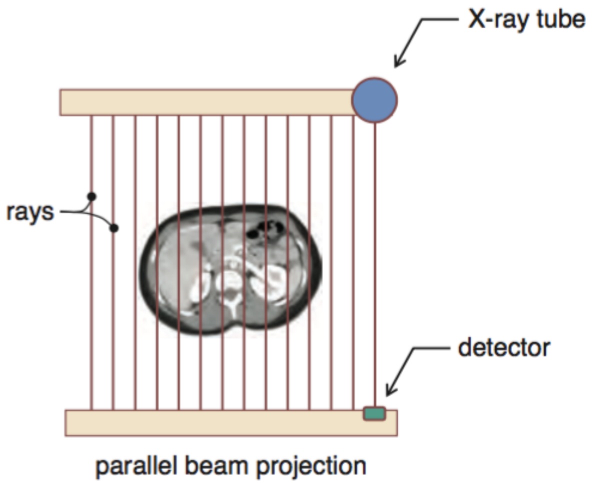


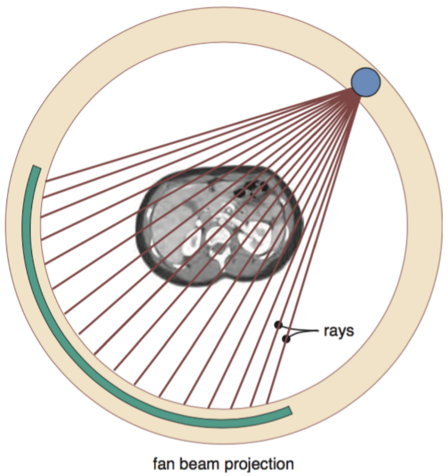


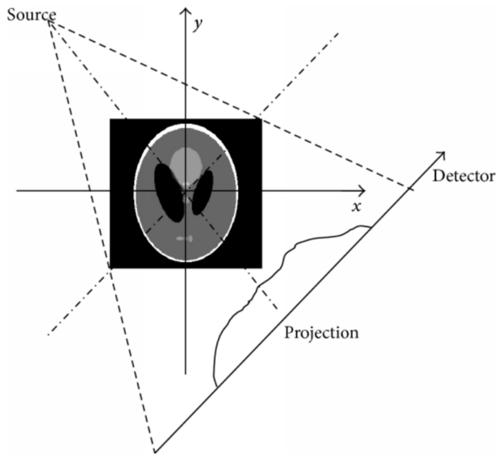
Advanced acquisition techniques (2/3)

Example: fan-beam geometry

- X-rays are *no longer parallel*







- Equations need to be rewritten in this **new coordinate system**!

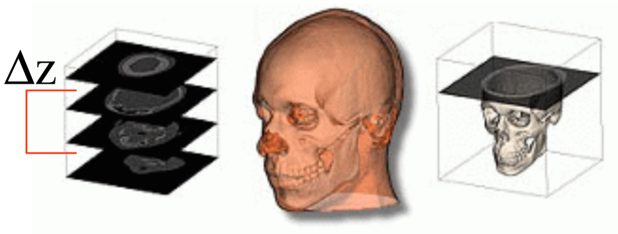
Also the **detector geometry** changes the underlying math

- *Linear* vs *curved* detectors
- *Multiple rows* of detectors

Imaging in 3D

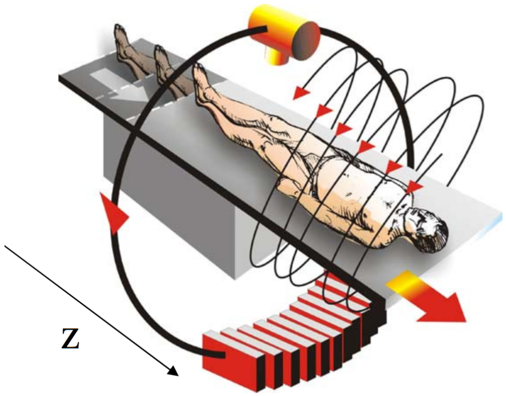
Sequential CT

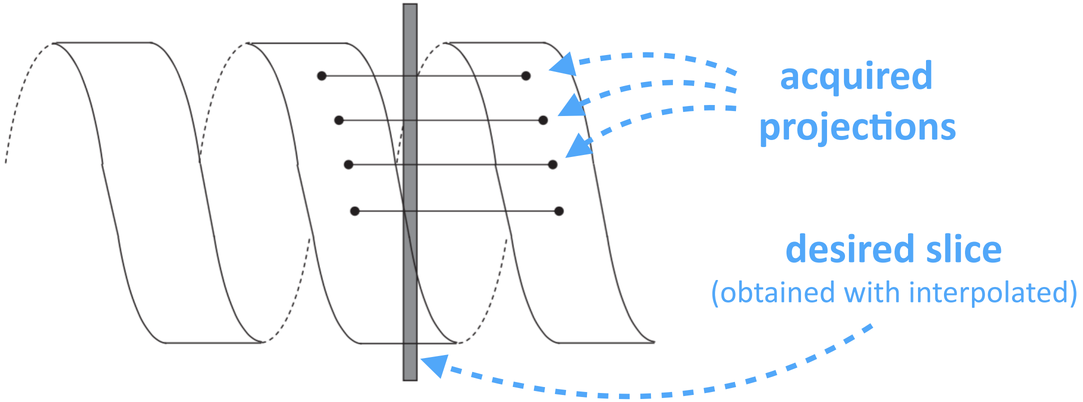
- advance table with patient (z direction) **after** each full 2D slice acquisition
- stop-motion shakes the patient ➜ motion artifacts
- very slow process



Spiral/helical CT

- advance table with patient *continuously* (z grows) **while** the X-ray source and detectors rotate (φ grows)
- very fast and stable
- allows reducing *tissue radiation doses*
- reconstruction process involves *interpolation* of data points not acquired





Summary

Pros

- As X-rays…
- …but allows 3D exams

Cons

- Ionizing radiation
- Higher radiation dose than X-rays

Applications

- As X-rays (e.g. bones fractures, lung cancer etc)
- Wider range of clinical conditions in almost any organ
- Very useful in *cardiac imaging*: especially in presence of pacemakers
  (so quick to acquire all projections that freezes the cardiac motion)
- *Liver imaging*: all projections at high-resolution in a single breath-hold